

Effects of recycle on heat and mass transfer between parallel-plate walls with equal fluxes

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(Received 12 August 1987 and in final form 9 February 1988)

Abstract—The effects of recycle at the ends on the heat and mass transfer between parallel-plate walls with equal fluxes are studied by a finite difference technique. Theoretical results show that the heat transfer efficiency can be augmented compared with that in a classical channel without recycle in the whole range of Graetz numbers. The compensation between the premixing and residence-time effects are used to explain the behavior of the fluid.

INTRODUCTION

THE HEAT and mass transfer in a duct has long been a problem of interest to engineers. Usually, one has to make assumptions to simplify the governing equations to obtain an analytical solution. For example, the Graetz problem concerns heat and mass transfer for fully developed flow at steady state with negligible axial conduction or diffusion [1, 2]. The extended Graetz problem considers furthermore, the influence of axial conduction or diffusion in the Graetz problem [3, 4]. The conjugated Graetz problem extends above problems in the single phase (or single stream) to those in the multiphase (or multistream) by conjugating the governing equations in each phase (or each stream) from the boundary conditions [1, 2, 5, 6]. A literature survey on the above problems showed that very few researchers investigated the effects of recycle of the fluid at the ends of a duct on heat and mass transfer. Actually, many separation processes and reactor designs in chemical engineering have been developed for countercurrent operation with internal or external recycle at the ends of the column [7-13]. In these processes, the recycle has large influences on the heat and mass transfer which in turn play a significant role in the design and operation of the equipment. Therefore, a thorough analysis of the effects of recycle on the transfer rate in these processes is necessary.

In the previous work [14], we have investigated theoretically and experimentally the influences of recycle on the heat and mass transfer through a parallel-plate channel with the first kind boundary condition. The results show that despite the advantage of the more uniform temperature or concentration of the fluid in the channel, the introduction of recycle may also enhance the transfer rate for large Graetz

number. However, as the Graetz number decreases, the heat and mass transfer decreases too, and eventually is less than that without recycle.

The aim of this paper is to extend the previous work from the first kind boundary condition to the second kind boundary condition. The purposes of this work are:

- (a) to study theoretically the effects of recycle on heat and mass transfer;
- (b) to see if the recycle of fluid is an effective means of augmenting the transfer efficiency for the whole range of Graetz numbers.

For simplicity, only the heat transfer between parallel-plate walls for homogeneous fluid in laminar flow at steady state is considered.

THEORETICAL FORMULATION

Consider a horizontal parallel-plate channel with thickness W , length L and infinite width as shown in Fig. 1. An impermeable plate with negligible thickness and thermal resistance is inserted between the walls to divide the channel into two parts with thicknesses ΔW and $(1-\Delta)W$, respectively. The fluid with volume flow rate V and temperature T_0 flows through the

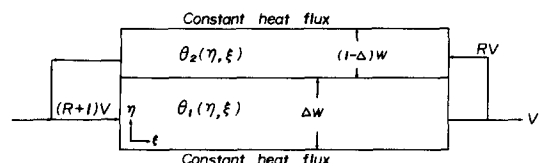


FIG. 1. Schematic diagram of the system.

NOMENCLATURE

B	channel width	V	input volume flow rate of feed
Gz	Graetz number, $U_b W^2 / \alpha L$	v	dimensionless velocity
k	thermal conductivity of fluid	W	thickness between the walls
L	channel length	x	longitudinal coordinate
Nu_1	local Nusselt number between the forward fluid and the lower wall, equation (11)	y	transversal coordinate.
Nu_2	local Nusselt number between the backward fluid and the upper wall, equation (12)	Greek symbols	
Nu_3	local Nusselt number between the forward and the backward fluids, equation (13)	α	thermal diffusivity
q	heat flux	Δ	ratio of thicknesses between forward flow channel and the whole channel
R	recycle ratio, recycle volume flow rate divided by input flow rate	η	dimensionless coordinate, y/W
T	fluid temperature	θ	dimensionless temperature, equation (4)
T_0	temperature of feed	ξ	dimensionless coordinate, x/LGz .
U	velocity distribution	Subscripts	
U_b	reference velocity, V/BW	1	forward flow channel
		2	backward flow channel
		b	bulk property.

channel. The volume flow rate of recycle RV is obtained externally or internally by using a jet, a conventional pump or an impeller at the end of the channel. Hence, the fluid will mix at the inlet of each channel.

By assuming constant physical properties of the fluid, equal heat fluxes applied to both walls, negligible end effects and axial conduction, and fully developed laminar flow existing in the channel, the velocity distribution and energy balance in dimensionless form may be formulated as

$$v_1(\eta) = 6(1+R) \left(\frac{\eta}{\Delta^2} - \frac{\eta^2}{\Delta^3} \right), \quad 0 \leq \eta \leq \Delta \quad (1)$$

$$v_2(\eta) = \frac{6R}{(1-\Delta)^3} [\eta^2 - (1+\Delta)\eta + \Delta], \quad \Delta \leq \eta \leq 1 \quad (2)$$

$$v_i(\eta) \frac{\partial \theta_i(\eta, \xi)}{\partial \xi} = \frac{\partial^2 \theta_i(\eta, \xi)}{\partial \eta^2}, \quad i = 1, 2 \quad (3)$$

where

$$\eta = \frac{y}{W}, \quad \xi = \frac{\alpha x}{U_b W^2} = \frac{x}{LGz}, \quad \theta_i = \frac{k(T_i - T_0)}{qW},$$

$$U_b = \frac{V}{BW}, \quad v_i = \frac{U_i}{U_b}. \quad (4)$$

The boundary conditions for solving the above equations are

$$\frac{\partial \theta_1(0, \xi)}{\partial \eta} = -1 \quad (5)$$

$$\frac{\partial \theta_2(1, \xi)}{\partial \eta} = 1 \quad (6)$$

$$\theta_1(\Delta, \xi) = \theta_2(\Delta, \xi) \quad (7)$$

$$\frac{\partial \theta_1(\Delta, \xi)}{\partial \eta} = \frac{\partial \theta_2(\Delta, \xi)}{\partial \eta} \quad (8)$$

$$\theta_1(\eta, 0) = \frac{-\int_{\Delta}^1 v_2(\eta) \theta_2(\eta, 0) d\eta}{R+1}, \quad 0 \leq \eta \leq \Delta \quad (9)$$

$$\theta_2(\eta, Gz^{-1}) = \frac{\int_0^{\Delta} v_1(\eta) \theta_1(\eta, Gz^{-1}) d\eta}{R+1}, \quad \Delta \leq \eta \leq 1. \quad (10)$$

Since the analytical solution of the above problem is still in question, we will use the numerical method to find the approximate solution. Once the temperature distribution of the fluid is found, the dimensionless wall temperatures $\theta_1(0, \xi)$ and $\theta_2(1, \xi)$ can be calculated. Alternatively, the local Nusselt numbers between the lower wall and the forward fluid, between the upper wall and the backward fluid, and between the fluids, respectively, can be computed from the relations

$$Nu_1 = \frac{h_1 W}{k} = \frac{1}{\theta_1(0, \xi) - \theta_{1,b}(\xi)} \quad (11)$$

$$Nu_2 = \frac{h_2 W}{k} = \frac{1}{\theta_2(1, \xi) - \theta_{2,b}(\xi)} \quad (12)$$

$$Nu_3 = \frac{h_3 W}{k} = \frac{\partial \theta_1(\Delta, \xi) / \partial \eta}{\theta_{2,b}(\xi) - \theta_{1,b}(\xi)} \quad (13)$$

in which the dimensionless bulk temperatures for both

Table 1. Dimensionless inlet and outlet bulk temperatures with the recycle ratio and the Graetz number as parameters for $\Delta = 0.5$

Gz	R					
	$\theta_{1,b}(0)$		$\theta_{1,b}(Gz^{-1}) = \theta_{2,b}(Gz^{-1})$		$\theta_{2,b}(0)$	
	0.5	5.0	0.5	5.0	0.5	5.0
1	3.285×10^{-1}	1.723	1.956	1.977	9.855×10^{-1}	2.0676
10^1	1.109×10^{-1}	1.810×10^{-1}	1.999×10^{-1}	1.985×10^{-1}	3.327×10^{-1}	2.172×10^{-1}
10^2	1.302×10^{-2}	1.804×10^{-2}	2.006×10^{-2}	1.983×10^{-2}	3.906×10^{-2}	2.164×10^{-2}
10^3	1.344×10^{-3}	1.806×10^{-3}	2.023×10^{-3}	1.985×10^{-3}	4.032×10^{-3}	2.167×10^{-3}

channels are calculated by

$$\theta_{1,b}(\xi) = \frac{\int_0^\Delta v_1(\eta)\theta_1(\eta, \xi) d\eta}{R+1} \tag{14}$$

$$\theta_{2,b}(\xi) = \frac{-\int_\Delta^1 v_2(\eta)\theta_2(\eta, \xi) d\eta}{R} \tag{15}$$

NUMERICAL SOLUTION

A finite difference technique was used to solve the above equations. In order to prevent the oscillation of the wall temperature at the entrance of each channel, the fully implicit formulation was applied to equation (3). The algorithm to solve the resultant algebraical equations was the same as that used in the previous work [14].

(1) Assume the interfacial temperature distribution to satisfy equation (7). It has been found that this temperature should be assumed to increase rapidly at the entrance to obtain convergence, e.g.

$$\begin{aligned} & [\theta_1(\Delta, \xi = Ih) - \theta_1(\Delta, 0)] \\ & = \begin{cases} \left\{ \frac{I[\theta_1(\Delta, Gz^{-1}) - \theta_1(\Delta, 0)]}{N} \right\}^{0.5}, & \text{for } I \leq \frac{N}{50} \\ \left\{ \frac{I[\bar{\theta}_1(\Delta, Gz^{-1}) - \theta_1(\Delta, 0)]}{N} \right\}, & \text{else} \end{cases} \end{aligned} \tag{16}$$

where $h = Gz^{-1} N^{-1}$ is the mesh size in the longitudinal coordinate ξ and N the number of the mesh size. For simplicity, the dimensionless inlet temperatures $\theta_1(\eta, 0)$ and $\theta_2(\eta, Gz^{-1})$ are regarded as $\theta_1(\Delta, 0)$ and $\theta_2(\Delta, Gz^{-1})$, respectively.

(2) Solve the resultant two sets of the tridiagonal matrix and check if the temperature distribution satisfies equations (8)–(10). The relative errors for the dimensionless temperatures $\theta_1(\Delta, 0)$, $\theta_2(\Delta, Gz^{-1})$ and heat flux at each interfacial point are used as the tolerances to test the convergence.

(3) Whenever the tolerance for each interfacial point is not satisfied, try a new profile of interfacial temperature as follows:

$$\theta_1(\Delta, 0)|_{\text{new}} = 0.5(\theta_1(\Delta, 0)|_{\text{old}} + \theta_{1,b}(0)) \tag{17}$$

$$\theta_1(\Delta, Gz^{-1})|_{\text{new}} = 0.5(\theta_1(\Delta, Gz^{-1})|_{\text{old}} + \theta_{1,b}(Gz^{-1})) \tag{18}$$

$$\begin{aligned} \theta_1(\Delta, \xi = Ih)|_{\text{new}} &= \theta_1(\Delta, \xi = Ih)|_{\text{old}} \\ &+ \frac{S}{4} \left\{ \frac{\partial \theta_2(\Delta, \xi = Ih)}{\partial \eta} - \frac{\partial \theta_1(\Delta, \xi = Ih)}{\partial \eta} \right\} \end{aligned} \tag{19}$$

where S is the mesh size in the transversal coordinate η and the five-point finite difference formula has been used to calculate the first-order partial derivatives. The iteration is repeated until every tolerance is within $\pm 0.1\%$.

The mesh points (200, 40) and (200, 80) have been used for large and small Graetz numbers in the direction (η, ξ) . These mesh sizes are good enough to obtain a convergence, e.g. for $Gz = 10^2$ and $R = 0.5$, increasing the mesh points from (200, 40) to (200, 50) results in the value of $\theta_{1,b}(Gz^{-1})$ from 2.006×10^{-2} to 1.993×10^{-2} . One may obtain the dimensionless outlet bulk temperature $\theta_{1,b}(Gz^{-1}) = 2Gz^{-1}$, which is independent of parameters R and Δ , by making an energy balance over the whole channel. Therefore, this temperature can also be used as an additional criterion to check the convergence. Some results were tabulated in Table 1 with the recycle ratio and the Graetz number as parameters for $\Delta = 0.5$. The numerical value for each $\theta_{1,b}(Gz^{-1})$ has been found with relative error less than 2.5% compared to the theoretical value. The local Nusselt numbers and wall temperatures are presented in Figs. 2–10, in which the results for the fluid in the same channel without the introduction of an impermeable plate and recycle (i.e. classical channel) are also shown for comparison [1].

RESULTS AND DISCUSSIONS

As shown in Table 1, decreasing the Graetz number for a fixed recycle ratio results in the increase of the residence time of the fluid in the channel, and hence the dimensionless temperatures $\theta_{2,b}(0)$ and $\theta_{1,b}(0)$ before and after mixing with the feed. On the other hand, for a fixed large Graetz number (say $Gz = 10^3$), increasing the recycle ratio will also decrease the residence time of the fluid, and also the dimensionless bulk temperature $\theta_{2,b}(0)$. However, it will essentially reduce the contribution of the feed, and lead to an increase of the dimensionless inlet bulk temperature

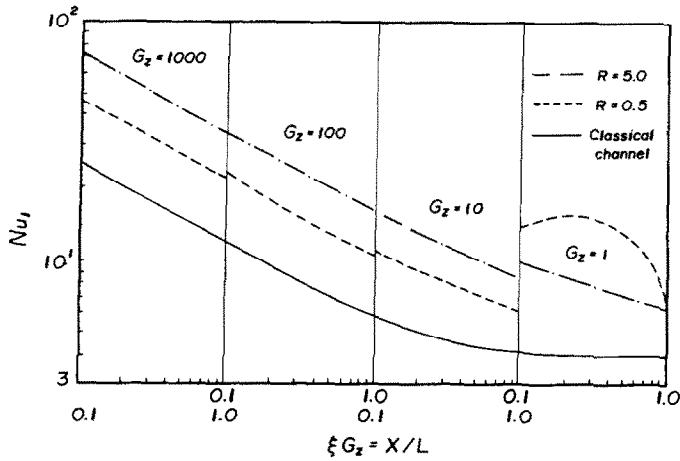


FIG. 2. Local Nusselt number between the forward fluid and the lower wall for $\Delta = 0.5$.

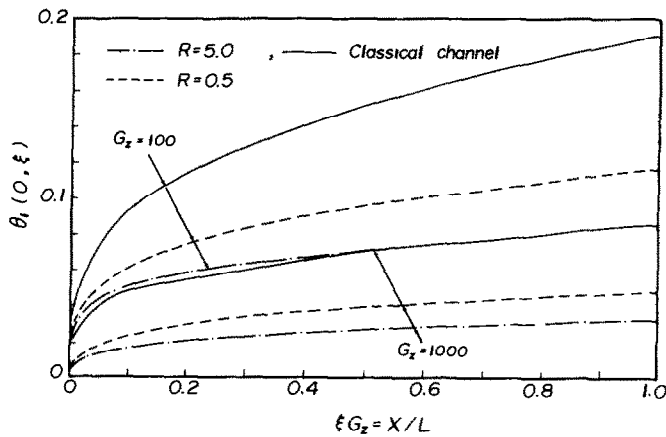


FIG. 3. Dimensionless lower wall temperature for $\Delta = 0.5$, $G_z = 10^2$ (or 10^3).

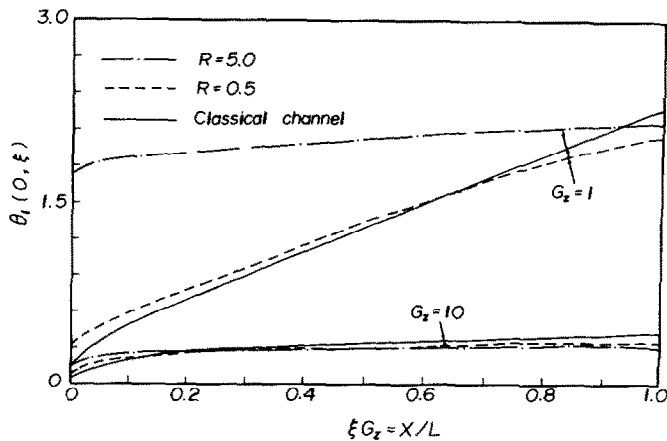


FIG. 4. Dimensionless lower wall temperature for $\Delta = 0.5$, $G_z = 1$ (or 10).

$\theta_{1,b}(0)$. We may conclude that the premixing effect of the feed increases when the recycle ratio rises or the Graetz number decreases. Consequently, the loss of residence-time effect is just compensated by the gain of premixing effect to make $\theta_{1,b}(Gz^{-1})$ equal to $2Gz^{-1}$. Moreover, inspection of the change of $\theta_{2,b}(Gz^{-1})$ to

$\theta_{2,b}(0)$ shows that the backward fluid receives heat from the upper wall and then releases part of the heat to the forward fluid. Hence, the heat absorbed by the forward fluid mainly comes from the lower wall. This behaviour is much the same as that in the classical channel, in which the position at $\eta = \Delta$ may be

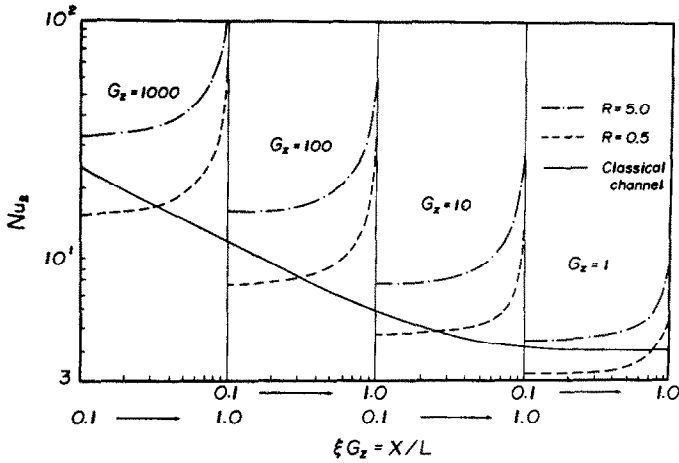


FIG. 5. Local Nusselt number between the backward fluid and the upper wall for $\Delta = 0.5$.

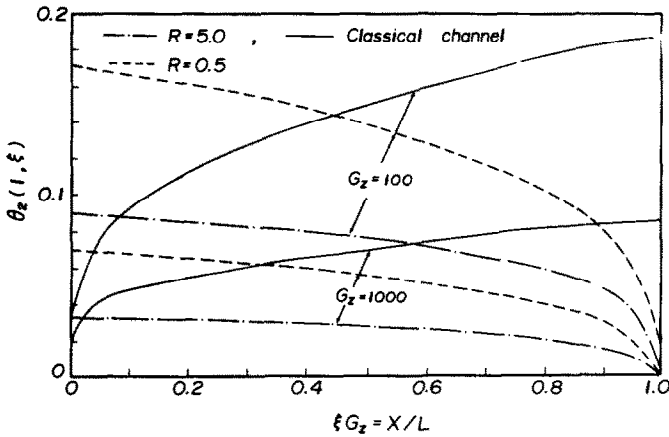


FIG. 6. Dimensionless upper wall temperature for $\Delta = 0.5$, $Gz = 10^2$ (or 10^3).

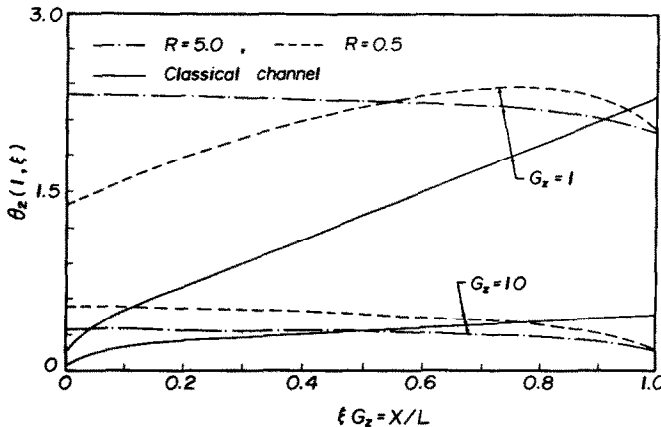


FIG. 7. Dimensionless upper wall temperature for $\Delta = 0.5$, $Gz = 1$ (or 10).

regarded as insulated. Accordingly, one may anticipate the same tendency of change of the local Nusselt number Nu_1 along the axis for both the improved channel and the classical channel as shown in Fig. 2. Furthermore, the greater the recycle ratio, the greater the value of Nu_1 compared with the classical channel,

because a smaller residence time of the fluid must associate with a higher heat transfer efficiency for a fixed outlet temperature. Hence, it gives a smaller wall temperature along the axis as shown in Fig. 3.

One may also use the concept of compensation between the residence-time effect and the premixing

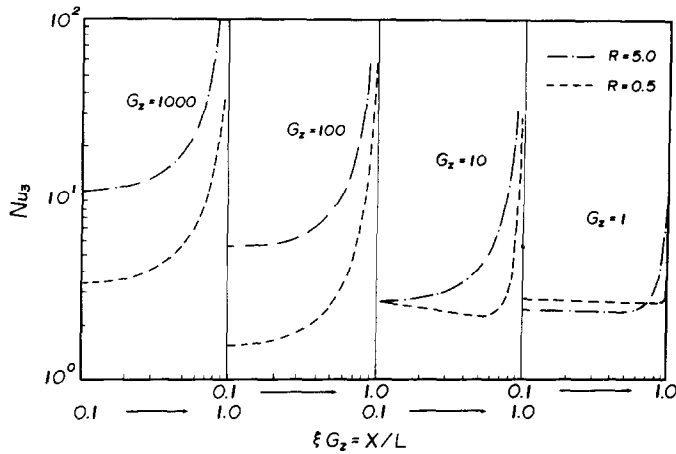


FIG. 8. Local Nusselt number between the forward and the backward fluids for $\Delta = 0.5$.

effect to explain the behaviour of $\theta_{1,b}(0)$ and $\theta_{1,b}(Gz^{-1})$ for the case of small Graetz numbers (say $Gz = 1.0$) as shown in Table 1. However, inspection of the change of $\theta_{2,b}(Gz^{-1})$ to $\theta_{2,b}(0)$ shows that the backward fluid releases almost the same heat as (for $R = 5.0$) or even much more heat than (for $R = 0.5$) the heat absorbed from the upper wall to the forward fluid. Accordingly, a large local Nusselt number Nu_1 , as shown in Fig. 2, does not mean a higher heat transfer efficiency, because the dimensionless bulk temperature $\theta_{1,b}(\xi)$ is not just influenced by the lower plate but also by the backward fluid. Yet, one may compare the results for both the improved channel and the classical channel from the wall temperature distribution as shown in Fig. 4. It is shown that increasing the recycle ratio will increase the dimensionless wall temperature and decrease the heat transfer efficiency. However, the heat transfer efficiency is still higher for the improved channel compared with that of the classical channel, because the maximum

wall temperature for the latter is more than that of the former.

Figure 5 gives the change of local Nusselt number Nu_2 along the axis with the recycle ratio and Graetz numbers as parameters. The behaviour is much the same as that in the classical channel if one follows along the flow of fluid. Moreover, one may use the same reasoning to explain the variation of Nu_2 with the recycle ratio as in the forward fluid. It is also better to investigate the advantage of recycle from the upper wall temperature distribution in Figs. 6 and 7. It is shown that the maximum wall temperature in the improved channel is always not more than that of the classical channel, especially as the recycle ratio is increased. Moreover, the value of $\theta_2(1, \xi)$ for $Gz = 1$ and $R = 0.5$ increases to a maximum and then decreases along the direction of flow. This behaviour differs from those for other cases, and can be attributed to the decrease in bulk temperature in the backward fluid.

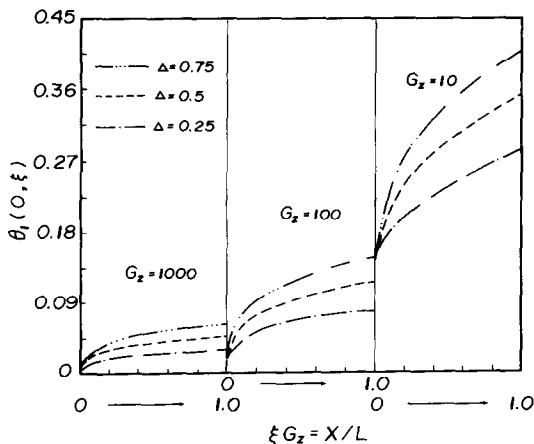


FIG. 9. Dimensionless lower wall temperature for $R = 0.5$ with Δ as parameter.

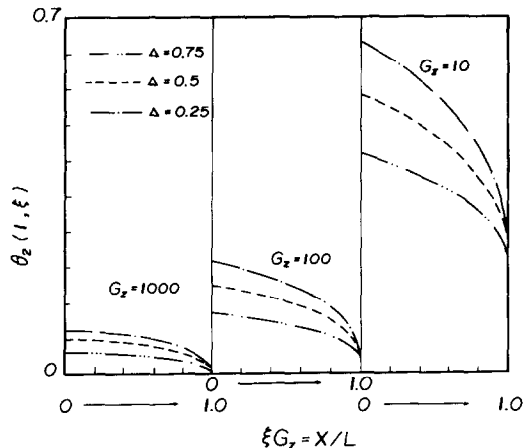


FIG. 10. Dimensionless upper wall temperature for $R = 0.5$ with Δ as parameter.

The local Nusselt numbers between the fluids are presented in Fig. 8. The value of Nu_3 increases along the axis due to the smaller difference between the bulk temperatures of the backward and the forward fluids. Again, the variation of Nu_3 with the recycle ratio is not easy to interpret, especially for small Graetz numbers. Moreover, Figs. 5 and 8 indicate that the Nusselt number tends to infinity at $X = L$. This result is apparently due to the combined effect of the assumed boundary conditions (7), (8) and (10), and the temperature of the singular point $\theta_2(1, Gz^{-1})$ has been set to $\theta_{2,b}(Gz^{-1})$ in the numerical solution. These force the denominator of equations (12) and (13) to tend toward zero. Thus, these results may be eliminated when the end effects are considered.

Figures 9 and 10 give the variation of dimensionless wall temperatures along the axis with the Graetz number and the thickness ratio as parameters for $R = 0.5$. It is shown that increasing the value of Δ results in an increase of $\theta_1(0, \xi)$ (and hence a decrease of heat transfer efficiency) and a decrease of $\theta_2(1, \xi)$ (and hence an increase of heat transfer efficiency), simultaneously. It is easy to interpret these results from the change of residence time of the fluids.

CONCLUSION

The effects of recycle on heat and mass transfer between parallel-plate walls with equal fluxes have been investigated theoretically by a finite difference method. The behaviour of the fluid may be described from the compensation between the residence-time effect and the premixing effect. Despite the advantage of the more uniform temperature in the channel with recycle, the introduction of the recycle can also enhance the heat transfer efficiency (and hence lower the maximum wall temperature) for the whole range of Graetz numbers. In general, the enhancement increases by increasing the values of Gz and R . Moreover, the greater the ratio Δ , the smaller the effect of the upper wall temperature and the greater that of the lower wall temperature.

EFFET DU RECYCLAGE SUR LE TRANSFERT DE CHALEUR ET DE MASSE ENTRE PAROIS PARALLELES PLANES AVEC FLUX EGAUX

Résumé—On étudie, par une technique aux différences finies, les effets du recyclage aux extrémités sur le transfert de chaleur et de masse entre parois planes parallèles. Les résultats théoriques montrent que l'efficacité du transfert thermique peut être augmentée, par rapport à celle du canal classique sans recyclage, dans tout le domaine des nombres de Graetz. La compensation des effets de prémélange et de temps de séjour permet d'expliquer le comportement du fluide.

EINFLUSS VON RÜCKSTRÖMUNGEN AUF DIE WÄRME- UND STOFFÜBERTRAGUNG ZWISCHEN PARALLELEN PLATTEN

Zusammenfassung—Mit Hilfe eines Finite-Differenzen-Verfahrens wird die Wärme- und Stoffübertragung zwischen zwei parallelen, gleichmäßig beaufschlagten Platten unter Berücksichtigung einer Rückströmung an den Enden untersucht. Theoretische Ergebnisse zeigen eine mögliche Erhöhung der Wärmeübertragung gegenüber derjenigen in einem herkömmlichen Kanal ohne Rückströmung für den gesamten Bereich von Graetz-Zahlen. Mit dem Ausgleich zwischen Vermischung und Effekten der Beharrungszeit wird das Verhalten des Fluids erklärt.

Acknowledgement—The authors wish to thank the Chinese National Science Council for financial aid with Grant No. NSC75-0402-E006-07.

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**ВЛИЯНИЕ КОНЦЕВЫХ ЭФФЕКТОВ НА ПРОЦЕССЫ ТЕПЛО-И МАССОПЕРЕНОСА
МЕЖДУ ПАРАЛЛЕЛЬНЫМИ ПЛАСТИНАМИ ПРИ ОДИНАКОВЫХ ПОТОКАХ**

Аннотация—Методом конечных разностей исследуется влияние концевых эффектов в канале из параллельных пластин на тепло-и массоперенос между ними при одинаковых потоках. Теоретические результаты показывают, что по сравнению с классическим случаем канала без концевых эффектов в рассматриваемом случае можно интенсифицировать теплоперенос во всем диапазоне значений числа Грэтца. Это объясняется компенсацией между эффектами предварительного перемешивания жидкости и времени пребывания ее в канале.